

CHAPTER 6

algebraic

EXPRESSIONS

iii

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6.1 CONCEPT OF EXPANDING BRACKETS

Expand a single pair of brackets

$w(x + y)$ can be expanded to become $wx + wy$ as shown below.

$$\begin{aligned}w(x + y) &= (w \times x) + (w \times y) \\ &= wx + wy\end{aligned}$$

RECALL

- $w \times x = wx$
- $w \times y = wy$
- wx and wy are unlike terms.
- unlike terms cannot be added together

Example 1

Expand the following algebraic expressions.

a) $r(s - 2t)$

b) $3e(2e - f + 4g)$

Solution:

a) $r(s - 2t) = (r \times s) + [r \times (-2t)]$

b) $3e(2e - f + 4g)$
 $= (3e \times 2e) + [3e \times (-f)] + (3e \times 4g)$
 $= 6e^2 - 3ef + 12eg$

TIPS

- $(+r) \times (+s) = +rs$
- $(+r) \times (-s) = -rs$
- $(-r) \times (+s) = -rs$
- $(-r) \times (-s) = +rs$

Exercise 6.1A

1. Expand the following algebraic expressions.

a) $p(q + r)$

b) $5(3x + y)$

c) $-w(x - y)$

d) $-3r(-t + 2s)$

e) $2q(2r - 3s + t)$

f) $\frac{1}{2}p(x + y)$

2. Expand the following expressions.

a) $-4m(-m - 3n)$

b) $4g(f/2 - 3g)$

c) $-n/2(2n + 8m)$

d) $4q/3(12p + 6q - 9r)$

Expand double pairs of brackets

$(w + x)(y + z)$ can be expanded to
 $wy + wz + xy + xz$ as shown below.

$$\begin{aligned}(w + x)(y + z) &= (w \times y) + (w \times z) + (x \times y) + (x \times z) \\ &= wy + wz + xy + xz\end{aligned}$$



Example 2

Expand the following algebraic expressions.

a) $(2a + b)(c + 3d)$

b) $(m + 4)(m - 3)$

Solution:

a) $(2a + b)(c + 3d)$

$$= (2a \times c) + (2a \times 3d) + (b \times c) + (b \times 3d)$$

$$= 2ac + 6ad + bc + 3bd$$



b) $(m + 4)(m - 3)$

$= (m \times m) + [m \times (-3)] + (4 \times m) + [4 \times (-3)]$

$= m^2 - \underbrace{3m + 4m}_{\text{Like terms}} - 12$

Like terms

$= m^2 + m - 12$



TIPS

- $(x \times w) = (w \times x) = wx$
- $(y \times w) = (w \times y) = wy$



Look at the products of $(a + b)$ and $(a - b)$.

$$(a + b)(a - b)$$

$$= (a \times a) + [a \times (-b)] + (b \times a) + [b \times (-b)]$$

$$= a^2 - ab + ab + b^2 = a^2 - b^2$$

Since $(a + b)(a - b) = (a - b)(a + b)$,

$(a - b)(a - b)$ is also equal to $a^2 - b^2$.

In general, $(a + b)(a - b) = (a - b)(a + b)$
 $= a^2 - b^2$

Example 3

Expand the following expressions.

a) $(a + 2b)(a - 2b)$

b) $(2p + 5)(2p - 5)$


Solution:

a) $(a + 2b)(a - 2b)$

$$= (a \times a) + [a \times (-2b)] + (2b \times a) + [2b \times (-2b)]$$


$$= a^2 - 2ab + 2ab - 4b^2$$

$$= a^2 - 4b^2 \leftarrow a^2 - 4b^2 = (a)^2 - (2b)^2$$



b) $(2p + 5)(2p - 5)$
 $= (2p \times 2p) + [2p \times (-5)] + (5 \times 2p) + [5 \times (-5)]$
 $= 4p^2 - 10p + 10p - 25$
 $= 4p^2 - 25$

$4p^2 - 25 = (2p)^2 - (5)^2$



NOTES

- $-10p$ and $10p$ are like terms.
- $-10p + 10p = 0$

Thus, $(a + b)(a - b)$ can be written as $a^2 - b^2$ directly.

Example 4

Expand the following expressions.

a) $(2x + 7)(2x - 7)$

b) $(3k - 2h)(3k + 2h)$

Solution:

a) $(2x + 7)(2x - 7)$

$$= (2x)^2 - (7)^2$$

$$= 4x^2 - 49$$

b) $(3k - 2h)(3k + 2h)$

$$= (3k)^2 - (2h)^2$$

$$= 9k^2 - 4h^2$$

NOTES

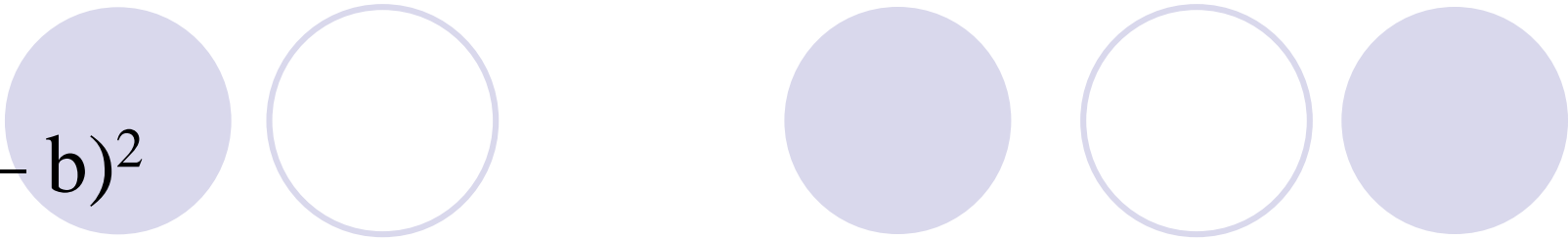
$$(a + b)(a - b) \neq a^2 + b^2$$

Look at the expansions of $(a + b)^2$ and $(a - b)^2$.

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= (a \times a) + (a \times b) + (b \times a) + (b \times b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

TIPS

- ab and ba are like terms.
- $ab + ba = 2ab$
- $-ab - ba = -2ab$


$$(a - b)^2$$

$$= (a - b)(a - b)$$

$$= (a \times a) + [a \times (-b)] + [(-b) \times a] + [(-b) \times (-b)]$$

$$= a^2 - ab - ba + b^2$$

$$= a^2 - 2ab + b^2$$

Thus,

$$(a + b)^2 = a^2 + 2ab + b^2$$

and

$$(a - b)^2 = a^2 - 2ab + b^2$$

Example 5

Expand the following expressions.

a) $(2m + 3n)^2$

b) $(4s - t)^2$

Solution:

$$(a + b)^2 = a^2 + 2ab + b^2$$

a) $(2m + 3n)^2 = (2m)^2 + 2(2m)(3n) + (3n)^2$
 $= 4m^2 + 12mn + 9n^2$

b) $(4s - t)^2 = (4s)^2 + 2(4s)(-t) + (-t)^2$
 $= 16s^2 - 8st + t^2$

$$(a - b)^2 = a^2 - 2ab + b^2$$



Exercise 6.1B

1. Expand the following expressions.

a) $(a + b)(c + 2d)$

b) $(2m - n)(2p - q)$

c) $(x + 7)(2y + 5)$

d) $(2p - 7)(q + 2)$

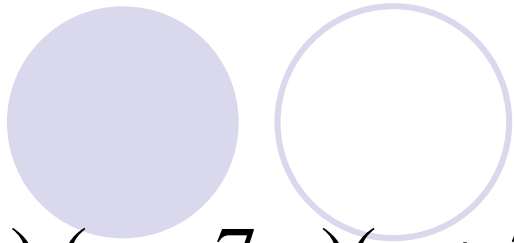
e) $(u - 2v)(3w + v)$

f) $(3k + 6)(k - 5)$

2. Expand the following expressions where each is the product of the sum and the difference of two terms.

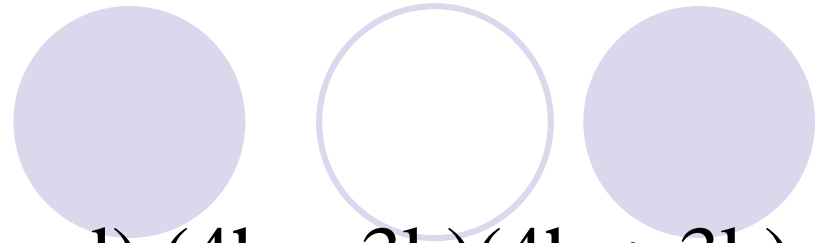
a) $(x + 3y)(x - 3y)$

b) $(f + 5g)(f - 5g)$



c) $(v - 7w)(v + 7w)$

e) $(5k + 2)(5k - 2)$



d) $(4h - 3k)(4h + 3k)$

f) $(7 + a)(7 - a)$

3. Expand the following expressions.

a) $(m + n)^2$

b) $(c - d)$

c) $(3w + 4x)^2$

d) $(5t - u)^2$

e) $(-y + 2)^2$

f) $(-2p - 5q)^2$

6.2 CONCEPT OF FACTORIZATION OF ALGEBRAIC EXPRESSIONS

State the factors of an algebraic term

A number can be expressed as the product of its factors.

$$\begin{aligned}18 &= 1 \times 18 \\ &= 2 \times 9 \\ &= 3 \times 6\end{aligned}$$

An algebraic term can be expressed as the product of its factors.

$$\begin{aligned}3ab &= 1 \times 3ab \\ &= 3 \times ab \\ &= a \times 3b \\ &= b \times 3a\end{aligned}$$

State the common factors and the highest common factor of several algebraic terms

- *Common factors of several Algebraic Terms*

Step 1: List all the factors of each algebraic term.

Step 2: Identify the factors which are common to all the terms.

Example 7

Find the common factors of the algebraic terms, $2ab$ and $4bc$.

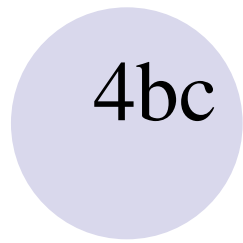
Solution:

$$\begin{aligned}2ab &= 1 \times 2ab \\ &= 2 \times ab \\ &= a \times 2b \\ &= b \times 2a\end{aligned}$$

The factors of $2ab$ are **1**, **2**, **a**, **b**, **2a**, **2b**, **ab** and **2ab**.

RECALL

- The common factors are factors which are common to all the numbers.
- The HCF is the highest common factor for all the numbers.
- Examples:
 - The factors of 8 are 1, 2, 4 and 8.
 - The factors of 12 are 1, 2, 3, 4, 6 and 12.
 - The common factors are 1, 2 and 4.
 - The HCF is 4.



$$= 1 \times 4bc$$

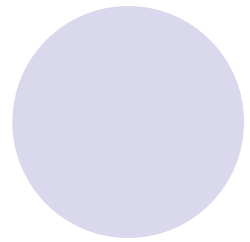
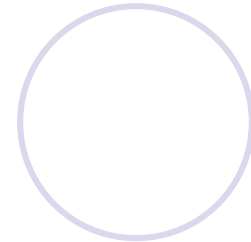
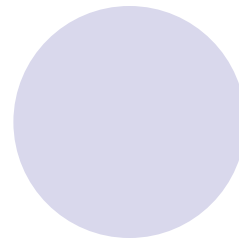
$$= 2 \times 2bc$$

$$= 4 \times bc$$

$$= b \times 4c$$

$$= c \times 4b$$

$$= 2b \times 2c$$



The factors of $4bc$ are **1**, **2**, 4, **b**, c , **2b**, $2c$, $4b$, $4c$, bc , $2bc$ and $4bc$.

Thus, the common factors of $2ab$ and $4bc$ are **1**, **2**, **b** and **2b**.

- *Highest Common Factor (HCF) of Several Algebraic Terms*

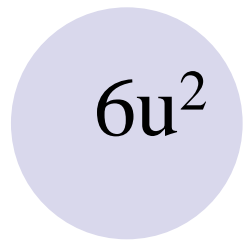
Example 8

Find the highest common factor of the algebraic terms, $3uv$ and $6u^2$.

Solution:

$$\begin{aligned}3uv &= 1 \times 3uv \\ &= 3 \times uv \\ &= u \times 3v \\ &= v \times 3u\end{aligned}$$

The factors of $3uv$ are **1**, **3**, **u**, **v**, **3u**, $3v$, uv and $3uv$.



$$= 1 \times 6u^2$$

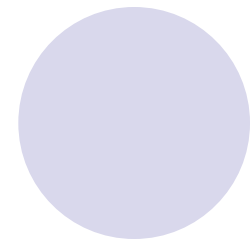
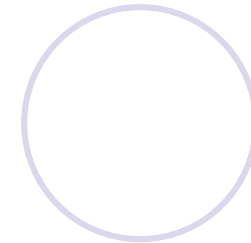
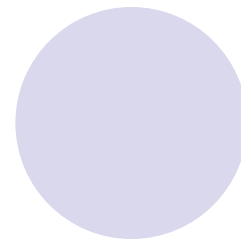
$$= 2 \times 3u^2$$

$$= 3 \times 2u^2$$

$$= 6 \times u^2$$

$$= u \times 6u$$

$$= 2u \times 3u$$



The factors of $6u^2$ are **1**, 2, **3**, 6, **u**, 2u, **3u**, 6u, u^2 , $2u^2$, $3u^2$ and $6u^2$.

The common factors of $3uv$ and $6u^2$ are **1**, **3**, **u** and **3u**.

Thus, the highest common factor of $3uv$ and $6u^2$ is **3u**.

ANOTHER WAY

Using the division method to find the HCF.

- Divide the terms by a common factor until no more common factor exists.

$$\begin{array}{r} 3 \\ u \end{array} \left\{ \begin{array}{l} \hline 3uv, 6u^2 \\ \hline uv, 2u^2 \\ \hline v, 2u \\ \hline \hline \end{array} \right.$$

The HCF of $3uv$ and $6u^2 = 3 \times u$
 $= 3u$

This is a faster way of determining HCF where many algebraic terms are involved.



Exercise 6.2A

Find the highest common factor (HCF) of the following algebraic terms.

1. $2f, 4g$

2. $4xy, 8yz$

3. $5g^2, 10gh$

4. n^2p, np^2

5. $8km^2, 12mn$

6. $ef^2, 2ef, 4efg$

7. $6abc^2, 12a^2bc, 18ab^2c$

Factorize algebraic expressions

NOTES

Expansion

$$p(q + r) = pq + pr$$

Factorization

Factorization is the reverse process of expansion.

Factorize $ab - ac$

Consider an expression $2ab - 4ac$.

$$\begin{array}{l} 2 \left\{ \begin{array}{l} \hline 2ab, 4ac \\ \hline \end{array} \right. \\ a \left\{ \begin{array}{l} ab, 2ac \\ \hline \end{array} \right. \\ \hline b, 2c \\ \hline \end{array}$$

$$\begin{aligned} \text{HCF} &= 2 \times a \\ &= 2a \end{aligned}$$

Thus, $2ab - 4ac$ can be expressed as the product of $2a$ and $(b - 2c)$.

NOTES

$$\begin{aligned} \frac{2ab - 4ac}{2a} &= \frac{2ab}{2a} - \frac{4ac}{2a} \\ &= b - 2c \end{aligned}$$

$$2ab - 4ac = 2a(b - 2c)$$

HCF

Final
quotient

Example 9

Factorize the following algebraic expressions.

a) $2pq + 4qr$

b) $mn - mn^2$

c) $3xy + 6yz - 9y^2$

Solution:

a) $2pq + 4qr = 2q(p + 2r)$

HCF

Final quotient

Check:

$$2q(p + 2r) = 2pq + 4qr$$

$$\begin{array}{r} 2 \overline{) 2pq, 4qr} \\ q \overline{) pq, 2qr} \\ \hline p, 2r \\ \hline \text{HCF} = 2q \end{array}$$

$$\text{b) } mn - mn^2 = mn(1 - n)$$

HCF

Final quotient

Check:

$$mn(1 - n) = mn - mn^2$$

$$m \left\{ \begin{array}{l} mn, mn^2 \end{array} \right.$$

$$n \left\{ \begin{array}{l} n, n^2 \end{array} \right.$$

$$1, n$$

$$\text{HCF} = mn$$

$$\text{c) } 3xy + 6yz - 9y^2 = 3y(x + 2z - 3y)$$

HCF

Final quotient

$$3 \left\{ \begin{array}{l} 3xy, 6yz, 9y^2 \end{array} \right.$$

$$y \left\{ \begin{array}{l} xy, 2yz, 3y^2 \end{array} \right.$$

$$x, 2z, 3y$$

$$\text{HCF} = 3y$$

Check:

$$3y(x + 2z - 3y) = 3xy + 6yz - 9y^2$$



Exercise 6.2B

Factorize the following algebraic expressions.

a) $2x + 8y$

b) $6d + 12$

c) $6uv - 3v$

d) $6m^2 - 4mn$

e) $10x^2y^2 - 5xy$

f) $36fg + 27g^2$

g) $3a - 6b + 9c$

h) $u^2 - uv + uw$

Factorize $a^2 + 2ab + b^2$

To factorize $a^2 + 2ab + b^2$, we can write $2ab$ as $ab + ab$.

The HCF of a^2 and ab is a . Thus, $a^2 + ab = a(a + b)$.
The HCF of ab and b^2 is b . Thus, $ab + b^2 = b(a + b)$.

$$\begin{aligned} \text{Hence, } a^2 + 2ab + b^2 &= \underbrace{a^2 + ab} + \underbrace{ab + b^2} \\ &= a(a + b) + b(a + b) \\ &= (a + b)(a + b) \\ &= (a + b)^2 \end{aligned}$$

$(a + b)$ is the common factor.

$$\begin{aligned} \text{Thus, } a^2 + 2ab + b^2 &= (a + b)(a + b) \\ &= (a + b)^2 \end{aligned}$$

Example 10

Factorize the following algebraic expressions.

a) $x^2 + 8x + 16$

b) $y^2 + 6yz + 9z^2$

Solution:

The HCF of x^2 and $4x$ is x . So $x^2 + 4x = x(x + 4)$.
The HCF of $4x$ and 16 is 4 . So $4x + 16 = 4(x + 4)$.

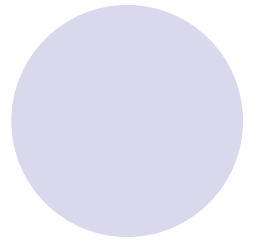
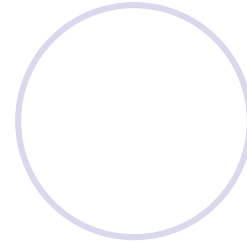
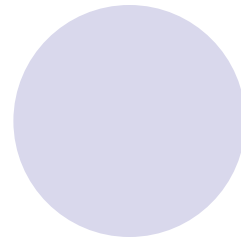
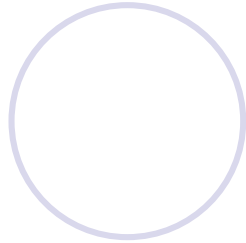
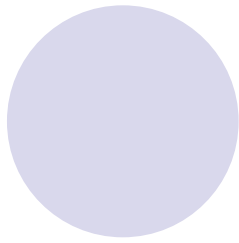
$$\begin{aligned} \text{a) } x^2 + 8x + 16 &= x^2 + 4x + 4x + 16 \leftarrow \\ &= x(x + 4) + 4(x + 4) \\ &= (x + 4)(x + 4) \\ &= (x + 4)^2 \leftarrow \end{aligned}$$

TIPS

- $8x = 4x + 4x$
- $6yz = 3yz + 3yz$

Check:

$$\begin{aligned} (x + 4)^2 &= (x)^2 + 2(x)(4) + (4)^2 \\ &= x^2 + 8x + 16 \end{aligned}$$



$$\begin{aligned} \text{b) } y^2 + 6yz + 9z^2 &= y^2 + 3yz + 3yz + 9z^2 \\ &= y(y + 3z) + 3z(y + 3z) \\ &= (y + 3z)(y + 3z) \\ &= (y + 3z)^2 \end{aligned}$$

Check:

$$\begin{aligned} (y + 3z)^2 &= (y)^2 + 2(y)(3z) + (3z)^2 \\ &= y^2 + 6yz + 9z^2 \end{aligned}$$



Exercise 6.2C

Factorize the following algebraic expressions.

1. $k^2 + 8k + 16$

2. $h^2 + 6h + 9$

3. $b^2 + 12b + 36$

4. $u^2 + 10uv + 25v^2$

5. $16y^2 + 8yz + z^2$

6. $4t^2 + 12t + 9$



Factorize $ab + ac + bd + cd$

Example 11

Factorize the following algebraic expressions.

a) $ax + 3a + 2x + 6$

b) $ab + by + ax + xy$

TIPS

- Group two terms in the expressions as one group.
- Find the HCF of each group.
- Divide each group by the HCF to get the final quotient.
- Make sure the two final quotients are the common factors.

Solution:

$$\text{a) } ax + 3a + 2x + 6$$

$$= (ax + 3a) + (2x + 6)$$

$$= a(\underline{x + 3}) + 2(\underline{x + 3})$$

$$= (x + 3)(a + 2)$$

Find the HCF of the expressions.

Common factor

$$\text{b) } ab + by + ax + xy$$

$$= (ab + by) + (ax + xy)$$

$$= b(\underline{a + y}) + x(\underline{a + y})$$

$$= (a + y)(b + x)$$

Find the HCF of the expressions.

Common factor



Exercise 6.2D

Factorize the following algebraic expressions.

1. $4ac + 2ab + 2cd + bd$

2. $st + 4tv + 3su + 12uv$

3. $2ab + 2ac + b + c$

4. $4x^2 - 2xy - 6xz + 3yz$



Factorize $a^2 - b^2$

Since factorization is the reverse process of expansion, $a^2 - b^2$ can be factorized as $(a + b)(a - b)$.

$a^2 - b^2$ is also known as the difference of two squares.

$$a^2 - b^2 = (a + b)(a - b)$$



Example 12

Factorize the following algebraic expressions.

a) $4x^2 - y^2$

b) $8p^2 - 18$

Solution:

$$\begin{aligned} \text{a) } 4x^2 - y^2 &= 2^2x^2 - y^2 \\ &= (2x)^2 - y^2 \\ &= (2x + y)(2x - y) \end{aligned}$$

$$\begin{aligned} \text{b) } 8p^2 - 18 &= 2(4p^2 - 9) \\ &= 2[(2p)^2 - 3^2] \\ &= 2(2p + 3)(2p - 3) \end{aligned}$$



Exercise 6.2E

Factorize the following algebraic expressions.

1. $p^2 - q^2$

2. $a^2 - 3^2$

3. $x^2 - 9$

4. $1 - 9t^2$

5. $16p^2 - 25q^2$

6. $81u^2 - 100$

7. $2x^2 - 18y^2$

8. $18gh^2 - 50g$

Factorize and simplify algebraic fractions

An algebraic fraction is one in which the numerator or the denominator or both are algebraic expressions. Examples:

$$\frac{3a^2}{6}, \frac{8}{4ef}, \frac{12gh^2k}{3gh}, \frac{9p + 6pq}{6 + 4q}, \frac{x^2 - y^2}{x^2 + xy}$$

Example 13

Simplify the following expressions.

a) $\frac{4ab^2}{6a^2c}$

b) $\frac{6pq}{9p^2qr}$

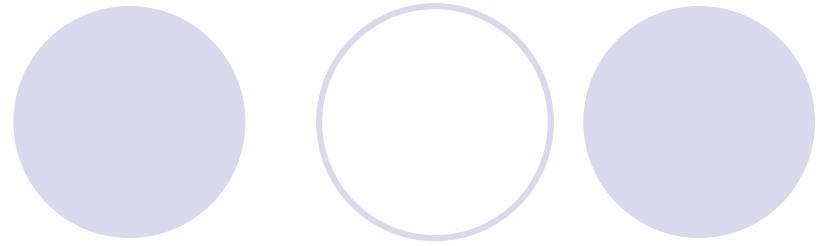
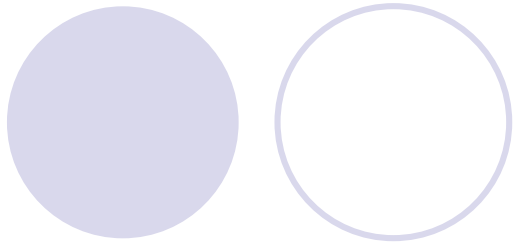
Solution:

a) $\frac{4ab^2}{6a^2c} = \frac{2a(2b)}{2a(3ac)}$

- The common factor of $4ab^2$ and $6a^2c$ is $2a$.
- Divide both numerator and denominator by $2a$.

$= \frac{2b^2}{3ac}$

No more common factor



$$\begin{aligned} \text{b) } \frac{6pq}{9p^2qr} &= \frac{3pq(2)}{3pq(3pr)} \\ &= \frac{2}{3pr} \end{aligned}$$

- The common factor of $6pq$ and $9p^2qr$ is $3pq$.
- Divide both numerator and denominator by $3pq$.

No more common factor

Example 14

Simplify the following expressions.

a) $\frac{2x + 10}{xy + 5y}$

b) $\frac{m^2 - 9}{mn - 3n}$

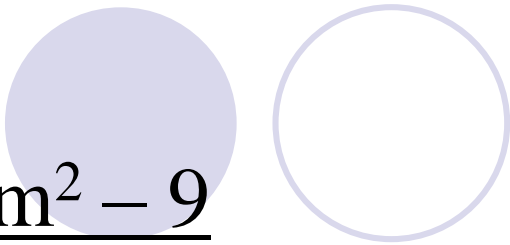
Solution:

a) $\frac{2x + 10}{xy + 5y}$

$$= \frac{2(x + 5)}{y(x + 5)}$$

$$= \underline{\underline{\frac{2}{y}}}$$

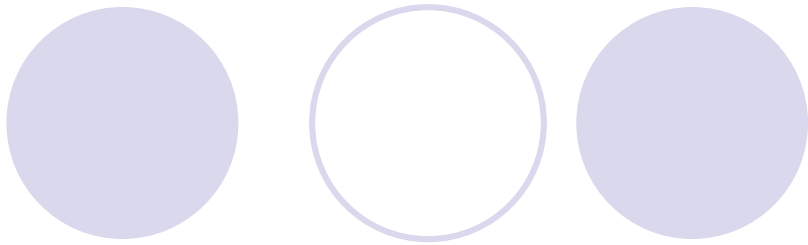
- Factorize both numerator and denominator.
- Divide by the common factor $(x + 5)$.



b) $\frac{m^2 - 9}{mn - 3n}$

$$= \frac{(m + 3)(m - 3)}{n(m - 3)}$$

$$= \frac{m + 3}{n}$$

- 
- Factorize both numerator and denominator.
 - Divide by the common factor $(m - 3)$.



Exercise 6.2F

1. Simplify the following algebraic fractions.

a) $\frac{5ab}{10bc}$


b) $\frac{6p}{18pr}$

c) $\frac{9pq}{n}$

d) $\frac{fg}{4fgh}$

e) $\frac{w^2u}{wu}$

f) $\frac{12b^2c^2}{abc}$



2. Simplify the following algebraic fractions to the lowest terms.

a) $\frac{12m + 3}{6p}$

b) $\frac{4a^2 - 12ab}{6ab}$

c) $\frac{pq}{pr - ps}$

d) $\frac{u + v}{5u + 5v}$

e) $\frac{s - t}{s^2 - t^2}$

f) $\frac{f^2 - 9}{f^2 - 3f}$

6.3 ADDITION AND SUBTRACTION OF ALGEBRAIC FRACTIONS

**Add or subtract two
algebraic fractions
with the same
denominator**

TIPS

- Add or subtract the numerators.
 - Retain the denominators.
- $+(a + b) = +a + b$
 $+(a - b) = +a - b$
 $-(a + b) = -a - b$
 $-(a - b) = -a + b$

Example 15

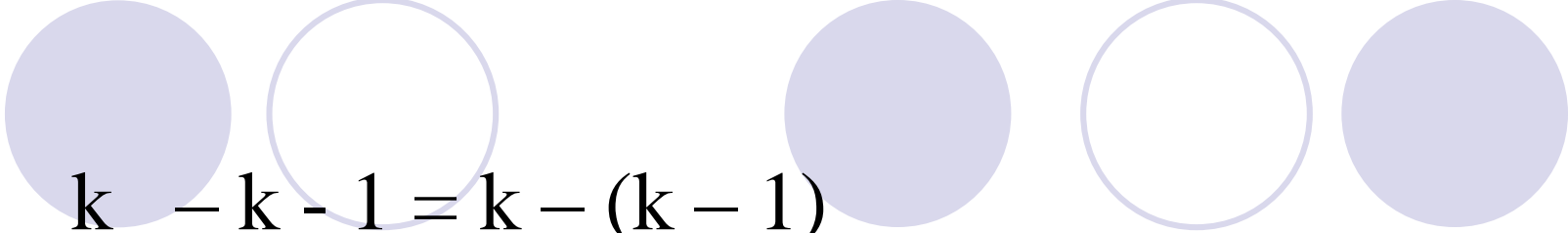
Simplify the following.

$$\text{a) } \frac{m + 3}{4} + \frac{2m + 1}{4}$$

$$\text{b) } \frac{k}{h + k} - \frac{k - 1}{h + k}$$

Solution:

$$\begin{aligned} \text{a) } \frac{m + 3}{4} + \frac{2m + 1}{4} &= \frac{(m + 3) + (2m + 1)}{4} \\ &= \frac{m + 3 + 2m + 1}{4} \\ &= \frac{3m + 4}{4} \end{aligned}$$



b) $\frac{k}{h+k} - \frac{k-1}{h+k} = \frac{k - (k-1)}{h+k}$

$$= \frac{k - k + 1}{h+k}$$

$$= \frac{1}{h+k}$$

Exercise 6.3A

Simplify the following to the lowest terms.

$$1. \frac{a}{3} + \frac{b}{3}$$

$$2. \frac{9x}{11} - \frac{2x}{11}$$

$$3. \frac{mn}{8} + \frac{3mn}{8}$$


$$4. \frac{10}{d^2} + \frac{3}{d^2}$$

$$5. \frac{3a + 1}{4b^2} + \frac{a + 2}{4b^2}$$

$$6. \frac{7w}{15wuv} - \frac{4w}{15wuv}$$

$$7. \frac{2p + 5}{3q} + \frac{p - 2}{3q}$$

$$8. \frac{b + 1}{2b + 1} + \frac{b + 1}{2b + 1}$$



Add or subtract two algebraic fractions with one denominator as a multiple of the other denominator

Example 16

Simplify the following to the lowest terms.

a)



Exercise 6.3B

Simplify the following to the lowest terms.

1. $\frac{a}{3} + \frac{a}{9}$

2. $\frac{2p}{5} - \frac{4p}{15}$

3. $\frac{y}{2} + \frac{y+2}{6}$

4. $\frac{b}{st} + \frac{d}{rst}$

5. $\frac{2k}{m+2} + \frac{4k}{3(m+2)}$

6. $\frac{f+g}{hk} - \frac{g-f}{2hk}$

Exercise 6.3C

1. Simplify each of the following.

a) $\frac{a}{2} + \frac{a}{5}$

b) $\frac{h}{m} - \frac{3}{n}$

c) $\frac{b}{a} + \frac{a}{b^2}$

d) $\frac{1}{2b} - \frac{b}{ac}$

e) $\frac{3y}{4b} + \frac{5q}{7}$

f) $\frac{3d}{a} - \frac{2a}{3d}$



2. Simplify each of the following algebraic fractions.

a) $\frac{x}{6} + \frac{y}{9}$

b) $\frac{p}{6} - \frac{q}{8}$

c) $\frac{h}{2k} - \frac{h}{k}$

d) $\frac{a}{rt} + \frac{b}{st}$

e) $\frac{b+d}{p} - \frac{c}{pq}$

f) $\frac{w}{ab} + \frac{w+u}{b}$

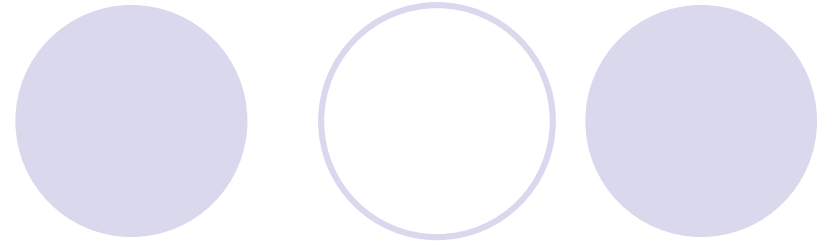
6.4 MULTIPLICATION AND DIVISION OF ALGEBRAIC FRACTIONS

Multiplication of two algebraic fractions

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

$$= \frac{ac}{bd}$$

Example 19



Exercise 6.4A

Find the product of the following.

$$1. \frac{p}{3} \times 5q$$

$$2. (w + u) \times \frac{x}{y}$$

$$3. \frac{2t}{v^2} \times \frac{w}{u^2}$$

$$4. \frac{5}{6g} \times (-h)$$

$$5. (x + y) \times \frac{z}{x^2 + 1}$$

$$6. \frac{4}{p - q} \times \frac{p + q}{5}$$

Exercise 6.4B

Find the quotient of the following.

$$1. \frac{m}{3} \div 5n$$

$$2. \frac{m}{p} \div \frac{n}{qr}$$

$$3. \frac{b^2c}{d} \div \frac{a}{3c}$$

$$4. \frac{5}{6w} \div (-u)$$

$$5. (x + y) \div \frac{z + 1}{w}$$

$$6. \frac{s}{p - q} \div \frac{a}{b + d}$$

Exercise 6.4C

1. Find the product of the following.

a) $\frac{p}{q} \times 3pq$

b) $15xy \times \frac{4}{20x}$

c) $\frac{uv}{wx} \times \frac{x^2}{u^2}$

d) $\frac{p}{qr} \times (-qr^2)$

e) $-\frac{rs^2}{4tu} \times \frac{uv}{r^2s}$

f) $\frac{4}{p-q} \times \frac{p^2 - q^2}{5}$

2. Divide the following.

a) $\frac{m}{3} \div mn$

b) $xyz \div \frac{wx}{y}$

c) $\frac{u^2}{6w} \div (-u)$

d) $(ax + ay) \div \frac{x + y}{w}$

e) $\frac{s}{p^2 - q^2} \div \frac{a}{p - q}$

f) $\frac{a + b}{c - d} \div \frac{a^2 - b^2}{3(c - d)}$

SUMMARY

ALGEBRAIC EXPRESSIONS III

Expansion

- The product of an algebraic term and an algebraic expression $a(b + c) = ab + ac$
- The product of two algebraic expressions $(a + b)(c + d) = ac + ad + bc + cd$
 $(a + b)^2 = a^2 + 2ab + b^2$
 $(a - b)^2 = a^2 - 2ab + b^2$
 $(a + b)(a - b) = a^2 - b^2$

Factorization

- Factorize an algebraic term $2ab = 2 \times a \times b$
- Factorize algebraic expressions
 $ab + ac = a(b + c)$
 $a^2 - b^2 = (a + b)(a - b)$
 $a^2 + 2ab + b^2 = (a + b)^2$
 $a^2 - 2ab + b^2 = (a - b)^2$
 $ab + ac + bd + cd = (b + c)(a + d)$

Algebraic fractions

Both or either the numerator or the denominator is an algebraic term or algebraic expression.

$$\frac{b}{5}, \frac{3}{a}, \frac{a+b}{a}, \frac{b}{a+b}, \text{ and } \frac{c-d}{a+b}$$

